Bug in Borwein Integrals

Enigma n°3

2nd December 2022

Looking at th first few cases, it seems that $\int_{-\infty}^{+\infty} \prod_{k=0}^{n} \frac{\sin(x/(2k+1)}{x/(2k+1)} dx = \pi$ for all $n \in \mathbb{N}$.
It is in fact true that:

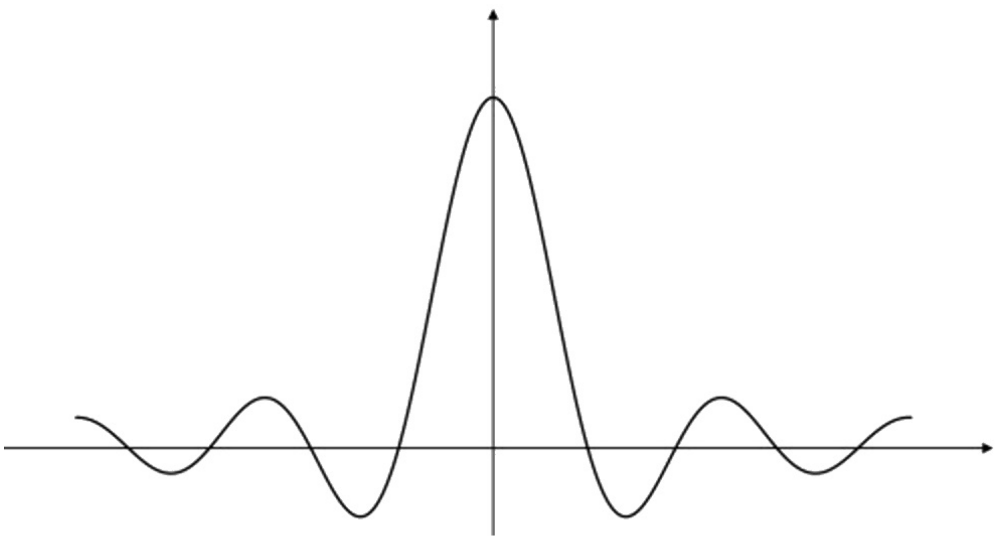
$$\int_{-\infty}^{+\infty} \frac{\sin(x)}{x} dx = \pi$$

$$\int_{-\infty}^{+\infty} \frac{\sin(x)}{x} \frac{\sin(x/3)}{x/3} dx = \pi$$

$$\int_{-\infty}^{+\infty} \frac{\sin(x)}{x} \frac{\sin(x/3)}{x/3} \frac{\sin(x/5)}{x/5} dx = \pi$$

However, it will eventually fail:

$$\int_{-\infty}^{+\infty} \prod_{k=0}^{100} \frac{\sin(x/(2k+1)}{x/(2k+1)} dx \neq \pi$$



Plot of $\frac{\sin(x)}{x}$

What is the first $n \in \mathbb{N}$ such that $\int_{-\infty}^{+\infty} \prod_{k=0}^{n} \frac{\sin(x/(2k+1)}{x/(2k+1)} dx \neq \pi$?