

Christmas Balls

Enigma n°8

14th December 2023

You are manufacturing magical Christmas decorations. All of them are n -dimensional balls (hyperballs, if you prefer), and they all have a radius of one. However, the decorations are magical, and may live in any dimension. You want to know what is the minimal and maximal volume of the decorations.



Christmas decorations in 1,2 and 3 dimensions.

The volume of a n -dimensional ball of radius R is given by: $\mathcal{V} = C_n R^n$. The constants C_n follow the rule $C_n = \frac{2\pi}{n} \cdot C_{n-2}$.

What is the highest value that C_n takes? Round it to 10^{-2} .

If the answer is $\frac{123456}{10^5}$, then submit "1.23".

Bonus: What is $\lim_{n \rightarrow +\infty} C_n$?